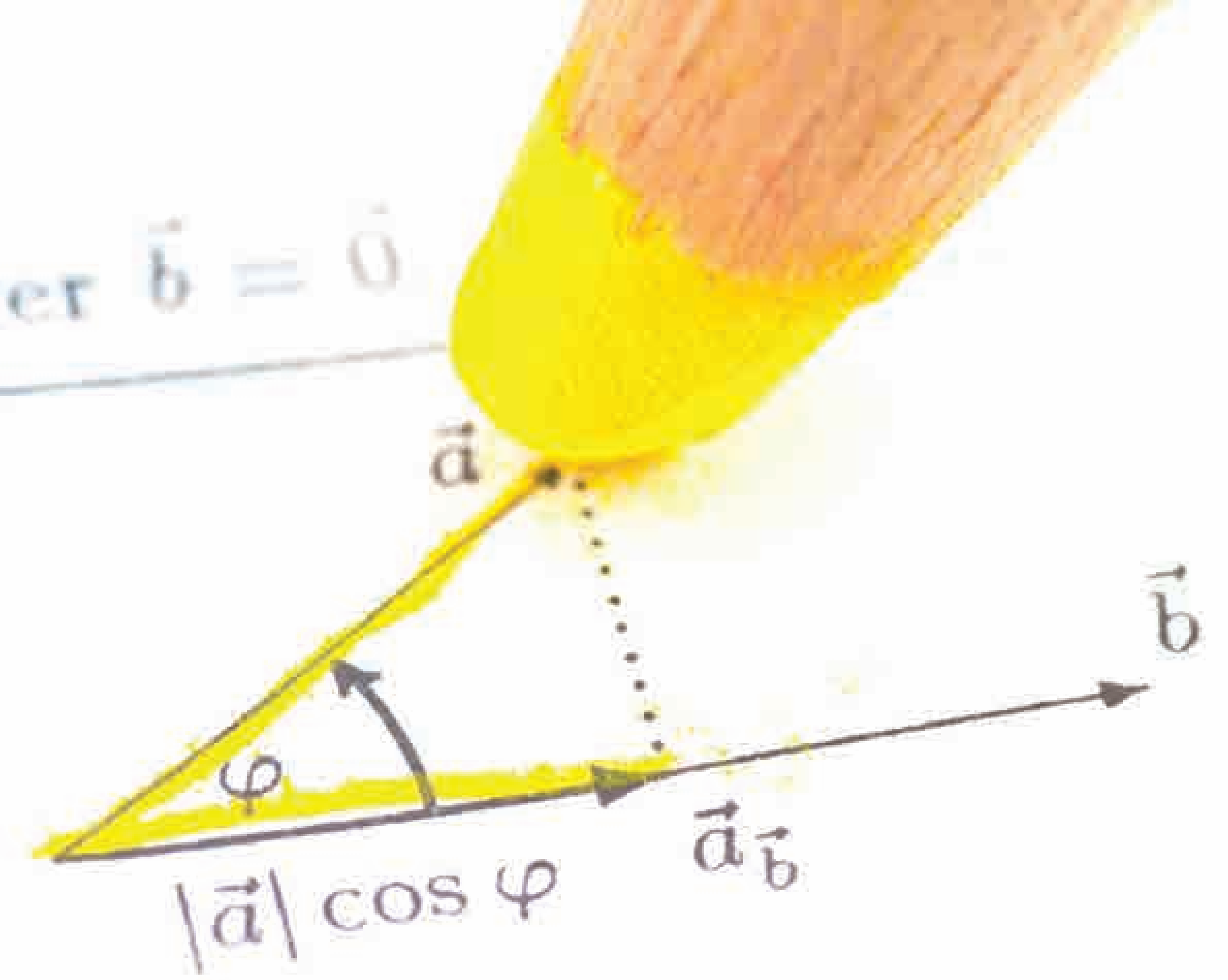

A2

Fundamental Theory

Network Protection & Automation Guide

Life Is On | Schneider Electric

Chapter A2

Fundamental Theory

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1. Introduction

The Protection Engineer is concerned with limiting the effects of disturbances in a power system. These disturbances, if allowed to persist, may damage plant and interrupt the supply of electric energy. They are described as faults (short and open circuits) or power swings, and result from natural hazards (for instance lightning), plant failure or human error.

To facilitate rapid removal of a disturbance from a power system, the system is divided into 'protection zones'. Relays monitor the system quantities (current, voltage) appearing in these zones; if a fault occurs inside a zone, the relays operate to isolate the zone from the remainder of the power system.

The operating characteristic of a relay depends on the energising quantities fed to it such as current or voltage, or various combinations of these two quantities, and on the manner in which the relay is designed to respond to this information. For example, a directional relay characteristic would be obtained by designing the relay to compare the phase angle between voltage and current at the relaying point. An impedance-measuring characteristic, on the other hand, would be obtained by designing the relay to divide voltage by current. Many other more complex relay characteristics may be obtained by supplying various combinations of current and voltage to the relay. Relays may also be designed to respond to other system quantities such as frequency, power, etc.

In order to apply protection relays, it is usually necessary to know the limiting values of current and voltage, and their relative phase displacement at the relay location, for various types of short circuit and their position in the system. This normally requires some system analysis for faults occurring at various points in the system.

The main components that make up a power system are generating sources, transmission and distribution networks, and loads. Many transmission and distribution circuits radiate from key points in the system and these circuits are controlled by circuit breakers. For the purpose of analysis, the power system is treated as a network of circuit elements contained in branches radiating from nodes to form closed loops or meshes. The system variables are current and voltage, and in steady state analysis, they are regarded as time varying quantities at a single and constant frequency. The network parameters are impedance and admittance; these are assumed to be linear, bilateral (independent of current direction) and constant for a constant frequency.

2. Vector algebra

A vector represents a quantity in both magnitude and direction. In Figure A2.1 the vector OP has a magnitude $|Z|$ at an angle θ with the reference axis OX.

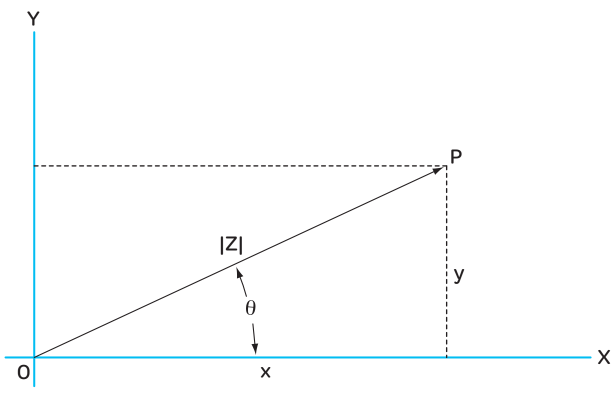

Figure A2.1: Vector OP

It may be resolved into two components at right angles to each other, in this case x and y. The magnitude or scalar value of vector Z is known as the modulus $|Z|$, and the angle θ is the argument, and is written as arg $\overline{Z}$.

The conventional method of expressing a vector $\overline{Z}$ is to write simply $|Z| \angle \theta$.

This form completely specifies a vector for graphical representation or conversion into other forms.

For vectors to be useful, they must be expressed algebraically. In Figure A2.1, the vector $\overline{Z}$ is the resultant of vectorially adding its components x and y; algebraically this vector may be written as:

$$\overline{Z} = x + jy \quad \text{...Equation A2.1}$$

where the operator j indicates that the component y is perpendicular to component x. In electrical nomenclature, the axis OC is the 'real' or 'in-phase' axis, and the vertical axis OY is called the 'imaginary' or 'quadrature' axis. The operator j rotates a vector anti-clockwise through 90°. If a vector is made to rotate anti-clockwise through 180°, then the operator j has performed its function twice, and since the vector has reversed its sense, then:

2. Vector algebra

$j \times j$ or $j^2 = -1$

whence $j = \sqrt{-1}$

The representation of a vector quantity algebraically in terms of its rectangular co-ordinates is called a 'complex quantity'. Therefore, $x + jy$ is a complex quantity and is the rectangular form of the vector $|Z| \angle \theta$ where:

$$\left.\begin{aligned} |Z| &= \sqrt{(x^2 + y^2)} \\ \theta &= tan^{-1} y/x \\ x &= |Z| cos\theta \\ y &= |Z| sin\theta \end{aligned}\right\} \quad \text{...Equation A2.2}$$

From Equations A2.1 and A2.2:

$$\bar{Z} = |Z|(cos\theta + jsin\theta) \quad \text{...Equation A2.3}$$

and since cos θ and sin θ may be expressed in exponential form by the identities:

$$sin\ \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$cos\ \theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

it follows that $\bar{Z}$ may also be written as:

$$\bar{Z} = |Z| e^{j\theta} \quad \text{...Equation A2.4}$$

Therefore, a vector quantity may also be represented trigonometrically and exponentially.

3. Manipulation of complex quantities

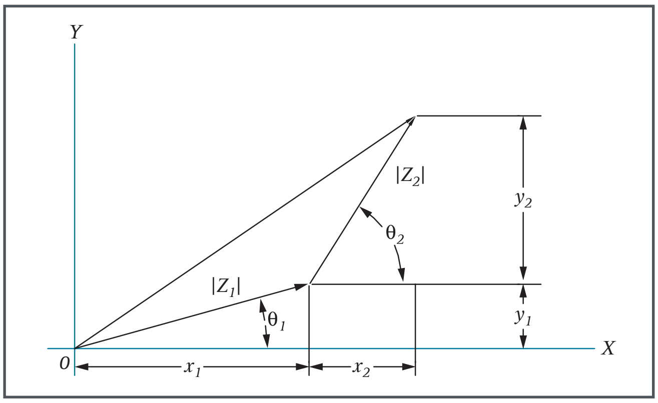

Figure A2.2:
Addition of vectors

Complex quantities may be represented in any of the four co-ordinate systems given below:

a. Polar $|Z| \angle \theta$

b. Rectangular $x + jy$

c. Trigonometric $|Z|(cos\theta + jsin\theta)$

d. Exponential $|Z| e^{j\theta}$

The modulus $|Z|$ and the argument θ are together known as 'polar co-ordinates', and x and y are described as 'cartesian co-ordinates'. Conversion between co-ordinate systems is easily achieved. As the operator j obeys the ordinary laws of algebra, complex quantities in rectangular form can be manipulated algebraically, as can be seen by the following:

$$\bar{Z}_1 + \bar{Z}_2 = (x_1 + x_2) + j(y_1 + y_2) \quad \text{...Equation A2.5}$$

$$\bar{Z}_1 - \bar{Z}_2 = (x_1 - x_2) + j(y_1 - y_2) \quad \text{...Equation A2.6 (see Figure A2.2)}$$

$$\left.\begin{aligned} \bar{Z}_1 \bar{Z}_2 &= |Z_1||Z_2| \angle \theta_1 + \theta_2 \\ \frac{\bar{Z}_1}{\bar{Z}_2} &= \frac{|Z_1|}{|Z_2|} \angle \theta_1 - \theta_2 \end{aligned}\right\} \quad \text{...Equation A2.7}$$

3.1 Complex variables

Some complex quantities are variable with, for example, time; when manipulating such variables in differential equations it is expedient to write the complex quantity in exponential form.

When dealing with such functions it is important to appreciate that the quantity contains real and imaginary components. If it is required to investigate only one component of the complex variable, separation into components must be carried out after the mathematical operation has taken place.

Example:

Determine the rate of change of the real component of a vector $|Z| \angle \omega t$ with time.

$$|Z| \angle \omega t = |Z|(cos\omega t + jsin\omega t) = |Z| e^{j\omega t}$$

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3. Manipulation of complex quantities

The real component of the vector is $|Z| \cos\omega t$. Differentiating $|Z| e^{j\omega t}$ with respect to time:

$$\frac{d}{dt}|Z|\, e^{j\omega t} = j\omega\, |Z|\, e^{j\omega t}$$

Separating into real and imaginary components:

$$\frac{d}{dt}\left(|Z|\, e^{j\omega t}\right) = |Z|\ (-\omega \sin\omega t\ +\ j\omega \cos\omega t)$$

Thus, the rate of change of the real component of a vector $|Z| \angle \omega t$ is:

$$-|Z|\omega \sin\omega t$$

3.2 Complex numbers

A complex number may be defined as a constant that represents the real and imaginary components of a physical quantity. The impedance parameter of an electric circuit is a complex number having real and imaginary components, which are described as resistance and reactance respectively.

Confusion often arises between vectors and complex numbers. A vector, as previously defined, may be a complex number. In this context, it is simply a physical quantity of constant magnitude acting in a constant direction. A complex number, which, being a physical quantity relating stimulus and response in a given operation, is known as a 'complex operator'. In this context, it is distinguished from a vector by the fact that it has no direction of its own.

Because complex numbers assume a passive role in any calculation, the form taken by the variables in the problem determines the method of representing them.

3.3 Mathematical operators

Mathematical operators are complex numbers that are used to move a vector through a given angle without changing the magnitude or character of the vector. An operator is not a physical quantity; it is dimensionless.

The symbol j, which has been compounded with quadrature components of complex quantities, is an operator that rotates a quantity anti-clockwise through 90°. Another useful operator is one which moves a vector anti-clockwise through 120°, commonly represented by the symbol a.

Operators are distinguished by one further feature; they are the roots of unity. Using De Moivre's theorem, the nth root of unity is given by solving the expression:

$$1^{1/n} = (\cos 2\pi m + j\sin 2\pi m)^{1/n}$$

where m is any integer. Hence:

$$1^{1/n} = \cos\frac{2\pi m}{n} + j\sin\frac{2\pi m}{n}$$

where m has values $1, 2, 3, \ldots (n-1)$

From the above expression j is found to be the 4th root and a the 3rd root of unity, as they have four and three distinct values respectively. Table A2.1 gives some useful functions of the a operator.

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j\frac{2P}{3}}$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{j\frac{4P}{3}}$$

$1 = 1 + j0 = e^{j0}$	$1 - a^2 = -j\sqrt{3a}$
$1 + a + a^2 = 0$	$a - a^2 = j\sqrt{3}$
$1 - a = j\sqrt{3a^2}$	$j = \frac{a - a^2}{\sqrt{3}}$

Table A2.1:
Properties of the a operator

4. Circuit quantities and conventions

Circuit analysis may be described as the study of the response of a circuit to an imposed condition, for example a short circuit. The circuit variables are current and voltage. Conventionally, current flow results from the application of a driving voltage, but there is complete duality between the variables and either may be regarded as the cause of the other.

When a circuit exists, there is an interchange of energy; a circuit may be described as being made up of 'sources' and 'sinks' for energy. The parts of a circuit are described as elements; a 'source' may be regarded as an 'active' element and a 'sink' as a 'passive' element. Some circuit elements are dissipative, that is, they are continuous sinks for energy, for example resistance. Other circuit elements may be alternately sources and sinks, for example capacitance and inductance. The elements of a circuit are connected together to form a network having nodes (terminals or junctions) and branches (series groups of elements) that form closed loops (meshes).

In steady state a.c. circuit theory, the ability of a circuit to accept a current flow resulting from a given driving voltage is called the impedance of the circuit. Since current and voltage are duals the impedance parameter must also have a dual, called admittance.

4.1 Circuit variables

As current and voltage are sinusoidal functions of time, varying at a single and constant frequency, they are regarded as rotating vectors and can be drawn as plan vectors (that is, vectors defined by two co-ordinates) on a vector diagram.

For example, the instantaneous value e, of a voltage varying sinusoidally with time is:

$$e = E_m \sin(\omega t + \delta) \qquad \text{...Equation A2.8}$$

where:

E_m is the maximum amplitude of the waveform

$\omega = 2\pi f$ is the angular velocity

δ is the argument defining the amplitude of the voltage at a time $t = 0$

At $t = 0$, the actual value of the voltage is $E_m \sin\delta$. So if E_m is regarded as the modulus of a vector, whose argument is δ, then $E_m \sin\delta$ is the imaginary component of the vector $|E_m|\angle\delta$.

Figure A2.3 illustrates this quantity as a vector and as a sinusoidal function of time.

The current resulting from applying a voltage to a circuit depends upon the circuit impedance. If the voltage is a sinusoidal function at a given frequency and the impedance is constant the current will also vary harmonically at the same frequency, so it can be shown on the same vector diagram as the voltage vector, and is given by the equation:

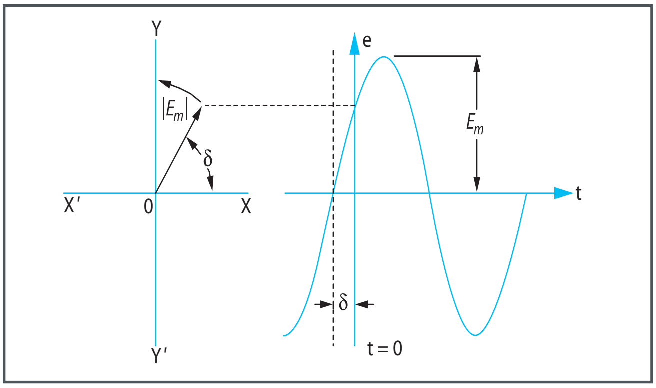

Figure A2.3: Representation of a sinusoidal function

$$i = \frac{|E_m|}{|Z|} \sin(\omega t + \delta - \phi) \qquad \text{...Equation A2.9}$$

where:

$$\left.\begin{aligned} |Z| &= \sqrt{R^2 + X^2} \\ X &= \left(\omega L - \frac{1}{\omega C}\right) \\ \phi &= \tan^{-1} X/R \end{aligned}\right\} \qquad \text{...Equation A2.10}$$

From Equations A2.9 and A2.10 it can be seen that the angular displacement ϕ between the current and voltage vectors and the current magnitude $|I_m| = |E_m|/|Z|$ is dependent upon the impedance $\bar{Z}$. In complex form the impedance may be written $\bar{Z} = R + jX$. The 'real component', R, is the circuit resistance, and the 'imaginary component', X, is the circuit reactance. When the circuit reactance is inductive (that is, $\omega L > 1/\omega C$), the current 'lags' the voltage by an angle ϕ, and when it is capacitive (that is, $1/\omega C > \omega L$) it 'leads' the voltage by an angle ϕ.

When drawing vector diagrams, one vector is chosen as the 'reference vector' and all other vectors are drawn relative to the reference vector in terms of magnitude and angle. The circuit impedance $|Z|$ is a complex operator and is distinguished from a vector only by the fact that it has no direction of its own. A further convention is that sinusoidally varying quantities are described by their 'effective' or 'root mean square' (r.m.s.) values; these are usually written using the relevant symbol without a suffix.

Thus:

$$\left.\begin{aligned} |I| &= |I_m|/\sqrt{2} \\ |E| &= |E_m|/\sqrt{2} \end{aligned}\right\} \qquad \text{...Equation A2.11}$$

The 'root mean square' value is that value which has the same heating effect as a direct current quantity of that value in the same circuit, and this definition applies to non-sinusoidal as well as sinusoidal quantities.

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4. Circuit quantities and conventions

4.2 Sign conventions

In describing the electrical state of a circuit, it is often necessary to refer to the 'potential difference' existing between two points in the circuit. Since wherever such a potential difference exists, current will flow and energy will either be transferred or absorbed, it is obviously necessary to define a potential difference in more exact terms. For this reason, the terms voltage rise and voltage drop are used to define more accurately the nature of the potential difference.

Voltage rise is a rise in potential measured in the direction of current flow between two points in a circuit. Voltage drop is the converse. A circuit element with a voltage rise across it acts as a source of energy. A circuit element with a voltage drop across it acts as a sink of energy. Voltage sources are usually active circuit elements, while sinks are usually passive circuit elements. The positive direction of energy flow is from sources to sinks.

Kirchhoff's first law states that the sum of the driving voltages must equal the sum of the passive voltages in a closed loop. This is illustrated by the fundamental equation of an electric circuit:

$$iR+\frac{Ldi}{dt}+\frac{1}{C}\int idt=e \qquad \text{...Equation A2.12}$$

where the terms on the left hand side of the equation are voltage drops across the circuit elements. Expressed in steady state terms Equation A2.12 may be written:

$$\sum \bar{E}=\sum \bar{I}\bar{Z} \qquad \text{...Equation A2.13}$$

and this is known as the equated-voltage equation [Ref A2.1: Power System Analysis].

It is the equation most usually adopted in electrical network calculations, since it equates the driving voltages, which are known, to the passive voltages, which are functions of the currents to be calculated.

In describing circuits and drawing vector diagrams, for formal analysis or calculations, it is necessary to adopt a notation which defines the positive direction of assumed current flow, and establishes the direction in which positive voltage drops and voltage rises act. Two methods are available: one, the double suffix method, is used for symbolic analysis; the other, the single suffix or diagrammatic method, is used for numerical calculations.

In the double suffix method the positive direction of current flow is assumed to be from node a to node b and the current is designated Iab. With the diagrammatic method, an arrow indicates the direction of current flow.

The voltage rises are positive when acting in the direction of current flow. It can be seen from Figure A2.4 that $\bar{E}_1$ and $\bar{E}_{an}$ are positive voltage rises and $\bar{E}_2$ and $\bar{E}_{bn}$ are negative voltage rises. In the diagrammatic method their direction of action is simply indicated by an arrow, whereas in the double suffix method, $\bar{E}_{an}$ and $\bar{E}_{bn}$ indicate that there is a potential rise in directions na and nb.

Voltage drops are also positive when acting in the direction of current flow. From Figure A2.4(a) it can be seen that $(\bar{Z}_1+\bar{Z}_2+\bar{Z}_3)\bar{I}$ is the total voltage drop in the loop in the direction of current flow, and must equate to the total voltage rise $\bar{E}_1-\bar{E}_2$.

In Figure A2.4(b), the voltage drop between nodes a and b designated $\bar{V}_{ab}$ indicates that point b is at a lower potential than a, and is positive when current flows from a to b. Conversely $\bar{V}_{ba}$ is a negative voltage drop.

Symbolically:

$$\left.\begin{aligned}\bar{V}_{ab}&=\bar{V}_{an}-\bar{V}_{bn}\\ \bar{V}_{ba}&=\bar{V}_{bn}-\bar{V}_{an}\end{aligned}\right\} \qquad \text{...Equation A2.14}$$

where n is a common reference point.

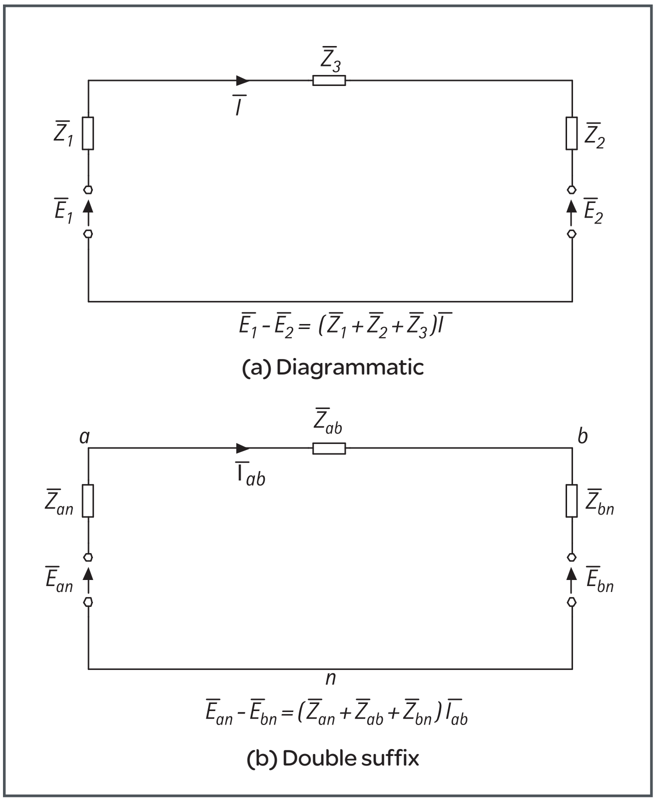

Figure A2.4:
Methods of representing a circuit

4.3 Power

The product of the potential difference across and the current through a branch of a circuit is a measure of the rate at which energy is exchanged between that branch and the remainder of the circuit. If the potential difference is a positive voltage drop, the branch is passive and absorbs energy. Conversely, if the potential difference is a positive voltage rise, the branch is active and supplies energy.

4. Circuit quantities and conventions

The rate at which energy is exchanged is known as power, and by convention, the power is positive when energy is being absorbed and negative when being supplied. With a.c. circuits the power alternates, so, to obtain a rate at which energy is supplied or absorbed, it is necessary to take the average power over one whole cycle.

If $e = E_m \sin(\omega t+\delta)$ and $i = I_m \sin(\omega t+\delta-\phi)$ then the power equation is:

$$p = ei = P[1-\cos 2(\omega t+\delta)]+Q\sin 2(\omega t+\delta)$$ *...Equation A2.15*

where:

$P = |E||I| \cos\phi$ and

$Q = |E||I| \sin\phi$

From Equation A2.15 it can be seen that the quantity P varies from 0 to $2P$ and quantity Q varies from $-Q$ to $+Q$ in one cycle, and that the waveform is of twice the periodic frequency of the current voltage waveform.

The average value of the power exchanged in one cycle is a constant, equal to quantity P, and as this quantity is the product of the voltage and the component of current which is 'in phase' with the voltage it is known as the 'real' or 'active' power.

The average value of quantity Q is zero when taken over a cycle, suggesting that energy is stored in one half-cycle and returned to the circuit in the remaining half-cycle.

Q is the product of voltage and the quadrature component of current, and is known as 'reactive power'. As P and Q are constants which specify the power exchange in a given circuit, and are products of the current and voltage vectors, then if $\bar{S}$ is the vector product $\bar{E}\bar{I}$ it follows that with $\bar{E}$ as the reference vector and ϕ as the angle between $\bar{E}$ and $\bar{I}$.

$$\bar{S} = P + jQ$$ *...Equation A2.16*

The quantity $\bar{S}$ is described as the 'apparent power', and is the term used in establishing the rating of a circuit. $\bar{S}$ has units of VA.

4.4 Single-phase and polyphase systems

A system is single or polyphase depending upon whether the sources feeding it are single or polyphase. A source is single or polyphase according to whether there are one or several driving voltages associated with it. For example, a three-phase source is a source containing three alternating driving voltages that are assumed to reach a maximum in phase order, A, B, C. Each phase driving voltage is associated with a phase branch of the system network as shown in Figure A2.5(a).

If a polyphase system has balanced voltages, that is, equal in magnitude and reaching a maximum at equally displaced time intervals, and the phase branch impedances are identical, it is called a 'balanced' system. It will become 'unbalanced' if any of the above conditions are not satisfied. Calculations using a balanced polyphase system are simplified, as it is only necessary to solve for a single phase, the solution for the remaining phases being obtained by symmetry.

The power system is normally operated as a three-phase, balanced, system. For this reason the phase voltages are equal in magnitude and can be represented by three vectors spaced 120° or 2π/3 radians apart, as shown in Figure A2.5(b).

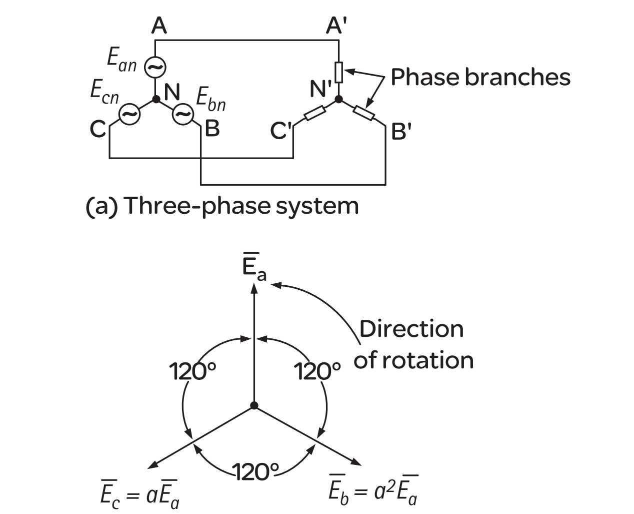

Figure A2.5:
Methods of representing a circuit

Since the voltages are symmetrical, they may be expressed in terms of one, that is:

$$\bar{E}_a = \bar{E}_a$$
$$\bar{E}_b = a^2 \bar{E}_a$$
$$\bar{E}_c = a \bar{E}_a$$ *...Equation A2.17*

where a is the vector operator $e^{j2p/3}$. Further, if the phase branch impedances are identical in a balanced system, it follows that the resulting currents are also balanced.

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5. Impedance notation

It can be seen by inspection of any power system diagram that:

a. several voltage levels exist in a system

b. it is common practice to refer to plant MVA in terms of per unit or percentage values

c. transmission line and cable constants are given in ohms/km

Before any system calculations can take place, the system parameters must be referred to 'base quantities' and represented as a unified system of impedances in either ohmic, percentage, or per unit values.

The base quantities are power and voltage. Normally, they are given in terms of the three-phase power in MVA and the line voltage in kV. The base impedance resulting from the above base quantities is:

$$Z_b = \frac{(kV)^2}{MVA}\ \Omega$$

...Equation A2.18

and, provided the system is balanced, the base impedance may be calculated using either single-phase or three-phase quantities.

The per unit or percentage value of any impedance in the system is the ratio of actual to base impedance values.

Hence:

$$\left.\begin{aligned} Z(p.u.) &= Z(\Omega) \times \frac{MVA_b}{(kV_b)^2} \\ Z(\%) &= Z(p.u.) \times 100 \end{aligned}\right\}$$

...Equation A2.19

Where: $MVA_b = base\ MVA$

$kV_b = base\ kV$

Simple transposition of the above formulae will refer the ohmic value of impedance to the per unit or percentage values and base quantities.

Having chosen base quantities of suitable magnitude all system impedances may be converted to those base quantities by using the equations given below:

$$\left.\begin{aligned} Z_{b2} &= Z_{b1} \times \frac{MVA_{b2}}{MVA_{b1}} \\ Z_{b2} &= Z_{b1} \times \left(\frac{kV_{b1}}{kV_{b2}}\right)^2 \end{aligned}\right\}$$

...Equation A2.20

where

suffix $b1$ denotes the value to the original base and $b2$ denotes the value to new base.

The choice of impedance notation depends upon the complexity of the system, plant impedance notation and the nature of the system calculations envisaged.

If the system is relatively simple and contains mainly transmission line data, given in ohms, then the ohmic method can be adopted with advantage. However, the per unit method of impedance notation is the most common for general system studies since:

a. impedances are the same referred to either side of a transformer if the ratio of base voltages on the two sides of a transformer is equal to the transformer turns ratio

b. confusion caused by the introduction of powers of 100 in percentage calculations is avoided

c. by a suitable choice of bases, the magnitudes of the data and results are kept within a predictable range, and hence errors in data and computations are easier to spot

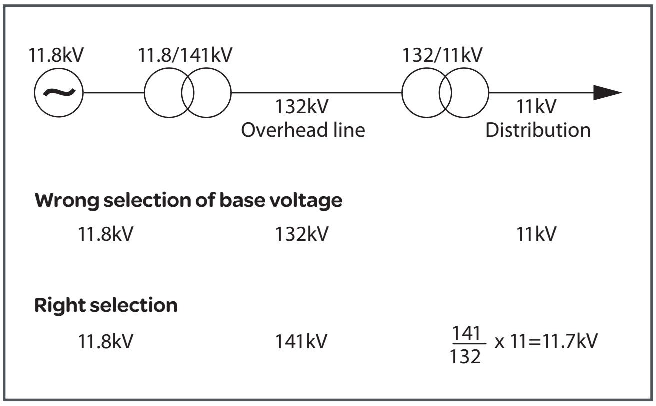

Figure A2.6:
Selection of base voltages

Most power system studies are carried out using software in per unit quantities. Irrespective of the method of calculation, the choice of base voltage, and unifying system impedances to this base, should be approached with caution, as shown in Figure A2.6.

From Figure A2.6 it can be seen that the base voltages in the three circuits are related by the turns ratios of the intervening transformers. Care is required as the nominal transformation ratios e.g. a 110/33kV (nominal) transformer may have a turns ratio of 110/34.5kV. Therefore, the rule for hand calculations is: 'to refer an impedance in ohms from one circuit to another multiply the given impedance by the square of the turns ratio (open circuit voltage ratio) of the intervening transformer'.

Where power system simulation software is used, the software normally has calculation routines built in to adjust transformer parameters to take account of differences between the nominal primary and secondary voltages and turns ratios. In this case, the choice of base voltages may be more conveniently made as the nominal voltages of each section of the power system. This approach avoids confusion when per unit or percent values are used in calculations in translating the final results into volts, amps, etc.

5. Impedance notation

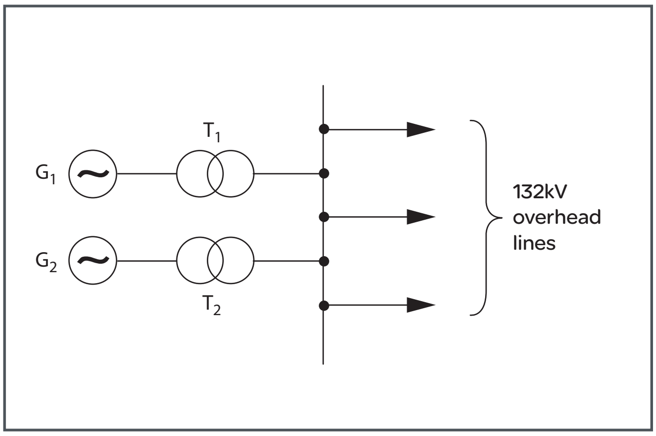

Figure A2.7:
Section of a power system

For example, in Figure A2.7, generators G_1 and G_2 have a sub-transient reactance of 26% on 66.6MVA rating at 11kV, and transformers T_1 and T_2 a voltage ratio of 11/145kV and an impedance of 12.5% on 75MVA. Choosing 100MVA as base MVA and 132kV as base voltage, find the percentage impedances to new base quantities.

a. Generator reactances to new bases are:

$$26 \times \frac{100}{66.6} \times \frac{(11)^2}{(132)^2} = 0.27\%$$

b. Transformer reactances to new bases are:

$$12.5 \times \frac{100}{75} \times \frac{(145)^2}{(132)^2} = 20.1\%$$

NOTE: *The base voltages of the generator and circuits are 11kV and 145kV respectively, that is, the turns ratio of the transformer. The corresponding per unit values can be found by dividing by 100, and the ohmic value can be found by using Equation A2.19.*

6. Basic circuit laws, theorems and network reduction

Most practical power system problems are solved by using steady state analytical methods. The assumptions made are that the circuit parameters are linear and bilateral and constant for constant frequency circuit variables. In some problems, described as initial value problems, it is necessary to study the behaviour of a circuit in the transient state. Such problems can be solved using operational methods. Again, in other problems, which fortunately are few in number, the assumption of linear, bilateral circuit parameters is no longer valid. These problems are solved using advanced mathematical techniques that are beyond the scope of this book.

6.1 Circuit laws

In linear, bilateral circuits, three basic network laws apply, regardless of the state of the circuit, at any particular instant of time. These laws are the branch, junction and mesh laws, due to Ohm and Kirchhoff, and are stated below, using steady state a.c. nomenclature.

Branch law

The current $\bar{I}$ in a given branch of impedance $\bar{Z}$ is proportional to the potential difference $\bar{V}$ appearing across the branch, that is, $\bar{V} = \bar{I}\bar{Z}$.

Junction law

The algebraic sum of all currents entering any junction (or node) in a network is zero, that is:

$$\sum \bar{I} = 0$$

Mesh law

The algebraic sum of all the driving voltages in any closed path (or mesh) in a network is equal to the algebraic sum of all the passive voltages (products of the impedances and the currents) in the components branches, that is:

$$\sum \bar{E} = \sum \bar{Z}\bar{I}$$

Alternatively, the total change in potential around a closed loop is zero.

6.2 Circuit theorems

From the above network laws, many theorems have been derived for the rationalisation of networks, either to reach a quick, simple, solution to a problem or to represent a complicated circuit by an equivalent. These theorems are divided into two classes: those concerned with the general properties of networks and those concerned with network reduction.

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6. Basic circuit laws, theorems and network reduction

Of the many theorems that exist, the three most important are given. These are: the Superposition Theorem, Thévenin's Theorem and Kennelly's Star/Delta Theorem.

Superposition theorem (general network theorem)

The resultant current that flows in any branch of a network due to the simultaneous action of several driving voltages is equal to the algebraic sum of the component currents due to each driving voltage acting alone with the remainder short-circuited.

Thévenin's theorem (active network reduction theorem)

Any active network that may be viewed from two terminals can be replaced by a single driving voltage acting in series with a single impedance. The driving voltage is the open-circuit voltage between the two terminals and the impedance is the impedance of the network viewed from the terminals with all sources short-circuited.

Kennelly's star/delta theorem (passive network reduction theorem)

Any three-terminal network can be replaced by a delta or star impedance equivalent without disturbing the external network. The formulae relating the replacement of a delta network by the equivalent star network is as follows (Figure A2.8):

$$\bar{Z}_{co} = \bar{Z}_{13} + \bar{Z}_{23} \Big/ \left(\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23} \right)$$

and so on.

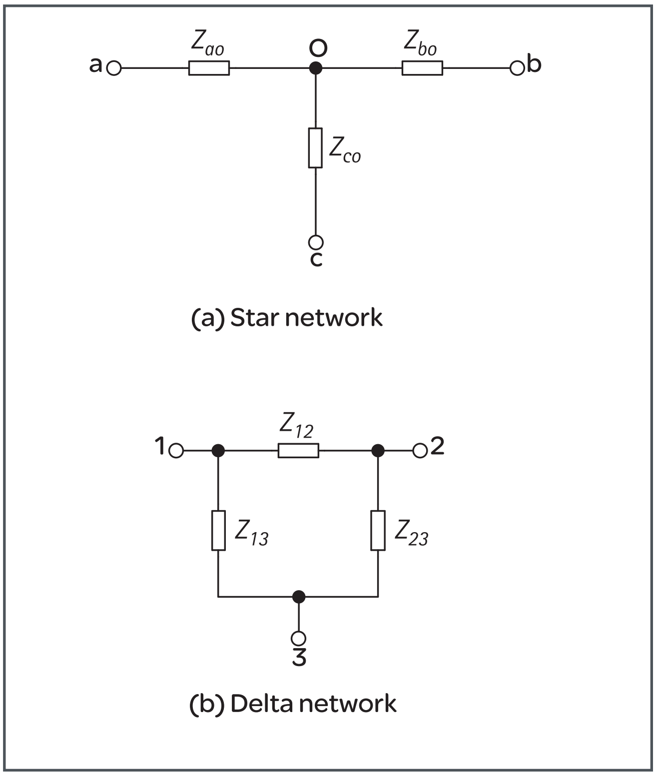

Figure A2.8 :
Star-delta network transformation

The impedance of a delta network corresponding to and replacing any star network is:

$$\bar{Z}_{12} = \bar{Z}_{ao} + \bar{Z}_{bo} + \frac{\bar{Z}_{ao}\,\bar{Z}_{bo}}{\bar{Z}_{co}}$$

and so on.

6.3 Network reduction

The aim of network reduction is to reduce a system to a simple equivalent while retaining the identity of that part of the system to be studied.

For example, consider the system shown in Figure A2.9. The network has two sources E' and E'', a line AOB shunted by an impedance, which may be regarded as the reduction of a further network connected between A and B, and a load connected between O and N. The object of the reduction is to study the effect of opening a breaker at A or B during normal system operations, or of a fault at A or B. Thus the identity of nodes A and B must be retained together with the sources, but the branch ON can be eliminated, simplifying the study. Proceeding, A, B, N, forms a star branch and can therefore be converted to an equivalent delta.

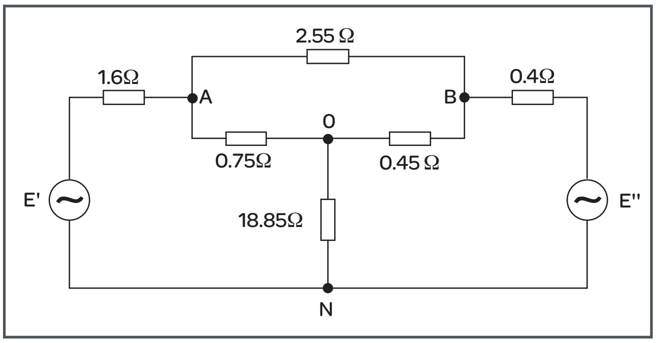

Figure A2.9:
Typical power system network

$$Z_{AN} = Z_{AO} + Z_{NO} + \frac{Z_{AO}\,Z_{NO}}{Z_{BO}} = 0.75 + 18.85 + \frac{0.75 \times 18.85}{0.45}$$
$$= 51\ \Omega$$

$$Z_{BN} = Z_{BO} + Z_{NO} + \frac{Z_{BO}\,Z_{NO}}{Z_{AO}} = 0.45 + 18.85 + \frac{0.45 \times 18.85}{0.75}$$
$$= 30.6\ \Omega$$

$$Z_{AB} = Z_{AO} + Z_{BO} + \frac{Z_{AO}\,Z_{BO}}{Z_{NO}} = 1.2\ \Omega \,(\text{since } Z_{NO} >>> Z_{AO} Z_{BO})$$

The network is now reduced as shown in Figure A2.10.

By applying Thévenin's theorem to the active loops, these can be replaced by a single driving voltage in series with an impedance as shown in Figure A2.11.

6. Basic circuit laws, theorems and network reduction

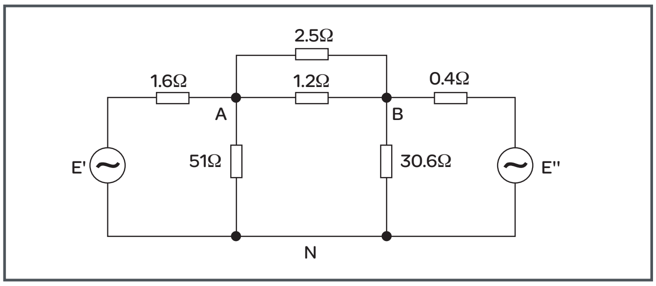

Figure A2.10:
Reduction using star/delta transformation

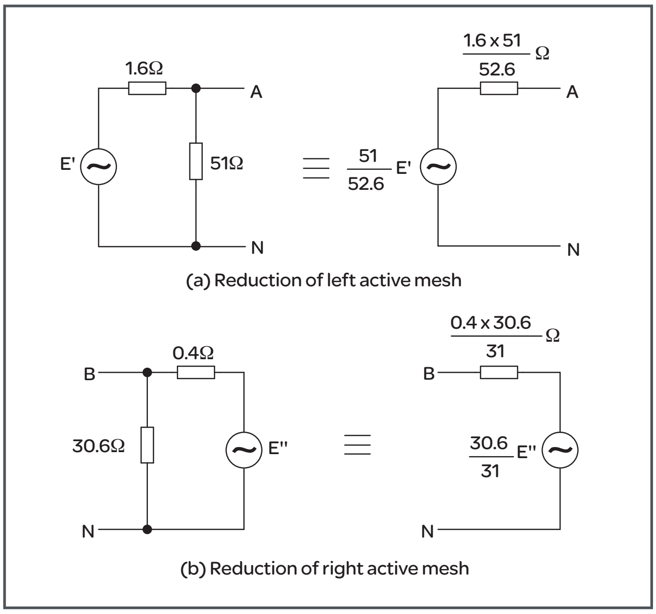

Figure A2.11:
Reduction of active meshes, Thévenin's Theorem

The network shown in Figure A2.9 is now reduced to that shown in Figure A2.12 with the nodes A and B retaining their identity. Further, the load impedance has been completely eliminated.

The network shown in Figure A2.12 may now be used to study system disturbances, for example power swings with and without faults.

Most reduction problems follow the same pattern as the example above. The rules to apply in practical network reduction are:

a. decide on the nature of the disturbance or disturbances to be studied

b. decide on the information required, for example the branch currents in the network for a fault at a particular location

c. reduce all passive sections of the network not directly involved with the section under examination

d. reduce all active meshes to a simple equivalent, that is, to a simple source in series with a single impedance

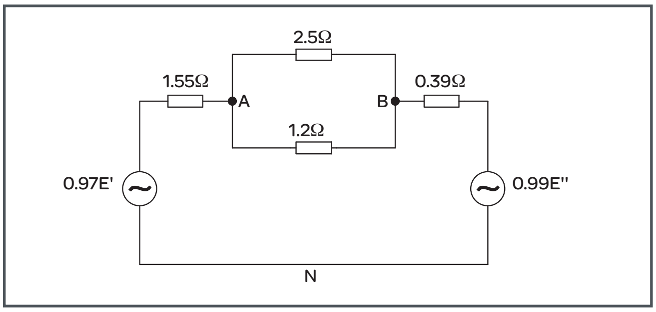

Figure A2.12:
Reduction of typical power system network

With the widespread availability of computer-based power system simulation software, it is now usual to use such software on a routine basis for network calculations without significant network reduction taking place. However, the network reduction techniques given above are still valid, as there will be occasions where such software is not immediately available and a hand calculation must be carried out.

In certain circuits, for example parallel lines on the same towers, there is mutual coupling between branches. Correct circuit reduction must take account of this coupling.

Three cases are of interest. These are:

a. two branches connected together at their nodes

b. two branches connected together at one node only

c. two branches that remain unconnected

Considering each case in turn:

a. consider the circuit shown in Figure A2.13(a). The application of a voltage V between the terminals P and Q gives:

$$V = I_a Z_{aa} + I_b Z_{ab}$$
$$V = I_a Z_{ab} + I_b Z_{bb}$$

where Ia and Ib are the currents in branches a and b, respectively and $I = I_a + I_b$, the total current entering at terminal P and leaving at terminal Q.

Solving for I_a and I_b:

$$I_b = \frac{(Z_{aa} - Z_{ab})V}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

from which

$$I_a = \frac{(Z_{bb} - Z_{ab})V}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

and

$$I = I_a + I_b = \frac{V(Z_{aa} + Z_{bb} - 2Z_{ab})}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

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6. Basic circuit laws, theorems and network reduction

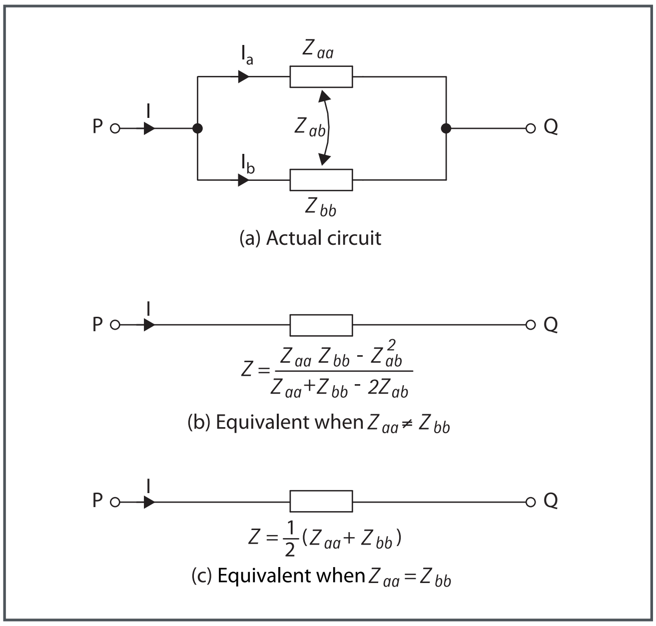

Figure A2.13:
Reduction of two branches with mutual coupling

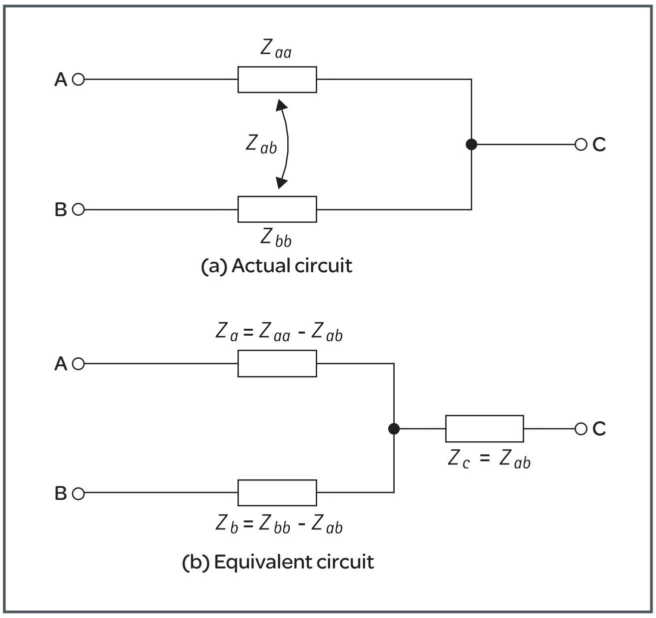

Figure A2.14:
Reduction of mutually-coupled branches with a common terminal

so that the equivalent impedance of the original circuit is:

$$Z = \frac{V}{I} = \frac{Z_{aa}Z_{bb} - Z_{ab}^2}{Z_{aa} + Z_{bb} - 2Z_{ab}} \quad \text{...Equation A2.21}$$

(Figure A2.13(b)), and, if the branch impedances are equal, the usual case, then:

$$Z = \frac{1}{2}\left(Z_{aa} + Z_{ab}\right) \quad \text{...Equation A2.22 (Figure A2.13(c))}$$

b. consider the circuit in Figure A2.14(a).

The assumption is made that an equivalent star network can replace the network shown. From inspection with one terminal isolated in turn and a voltage *v* impressed across the remaining terminals it can be seen that:

$$Z_a + Z_c = Z_{aa}$$
$$Z_b + Z_c = Z_{bb}$$
$$Z_a + Z_b = Z_{aa} + Z_{bb} + 2Z_{ab}$$

Solving these equations gives:

$$\left.\begin{aligned} Z_a &= Z_{aa} + Z_{ab} \\ Z_b &= Z_{bb} + Z_{ab} \\ Z_a &= -Z_{ab} \end{aligned}\right\} \quad \text{...Equation A2.23 (see Figure A2.14(b))}$$

c. consider the four-terminal network given in Figure A2.15(a), in which the branches 11' and 22' are electrically separate except for a mutual link. The equations defining the network are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

where $Z_{12}=Z_{21}$ and $Y_{12}=Y_{21}$, if the network is assumed to be reciprocal. Further, by solving the above equations it can be shown that:

$$\left.\begin{aligned} Y_{11} &= Z_{22}/\Delta \\ Y_{22} &= Z_{11}/\Delta \\ Y_{12} &= Z_{12}/\Delta \\ \Delta &= Z_{11}Z_{22} - Z_{12}^2 \end{aligned}\right\} \quad \text{...Equation A2.24}$$

There are three independent coefficients, namely Z_{12}, Z_{11}, Z_{22}, so the original circuit may be replaced by an equivalent mesh containing four external terminals, each terminal being connected to the other three by branch impedances as shown in Figure A2.15(b).

6. Basic circuit laws, theorems and network reduction

In order to evaluate the branches of the equivalent mesh let all points of entry of the actual circuit be commoned except node 1 of circuit 1, as shown in Figure A2.15(c). Then all impressed voltages except V_1 will be zero and:

$$I_1 = Y_{11} V_1$$
$$I_2 = Y_{12} V_1$$

If the same conditions are applied to the equivalent mesh, then:

$$I_1 = V_1\ Z_{11}$$
$$I_2 = -V_1/Z_{12} = -V_1/Z_{12}$$

These relations follow from the fact that the branch connecting nodes 1 and 1' carries current I_1 and the branches connecting nodes 1 and 2' and 1 and 2 carry current I_2. This must be true since branches between pairs of commoned nodes can carry no current.

By considering each node in turn with the remainder commoned, the following relationships are found:

$$Z_{11'} = 1/Y_{11}$$
$$Z_{22'} = 1/Y_{22}$$
$$Z_{12'} = -1/Y_{12}$$
$$Z_{12} = Z_{1'2'} = -Z_{21'} = -Z_{12'}$$

Hence:

$$\left.\begin{aligned} Z_{11'} &= \frac{Z_{11} Z_{22} - Z_{12}^2}{Z_{22}} \\ Z_{22'} &= \frac{Z_{11} Z_{22} - Z_{12}^2}{Z_{11}} \\ Z_{12'} &= \frac{Z_{11} Z_{22} - Z_{12}^2}{Z_{12}} \end{aligned}\right\} \quad \text{...Equation A2.25}$$

A similar but equally rigorous equivalent circuit is shown in Figure A2.15(d). This circuit [Ref A2.2: Equivalent Circuits I.] follows the fact that the self - impedance of any circuit is independent of all other circuits. Therefore, it need not appear in any of the mutual branches if it is lumped as a radial branch at the terminals. So putting Z_{11} and Z_{22} equal to zero in Equation A2.25, defining the equivalent mesh in Figure A2.15(b), and inserting radial branches having impedances equal to Z_{11} and Z_{22} in terminals 1 and 2, results in Figure A2.15(d).

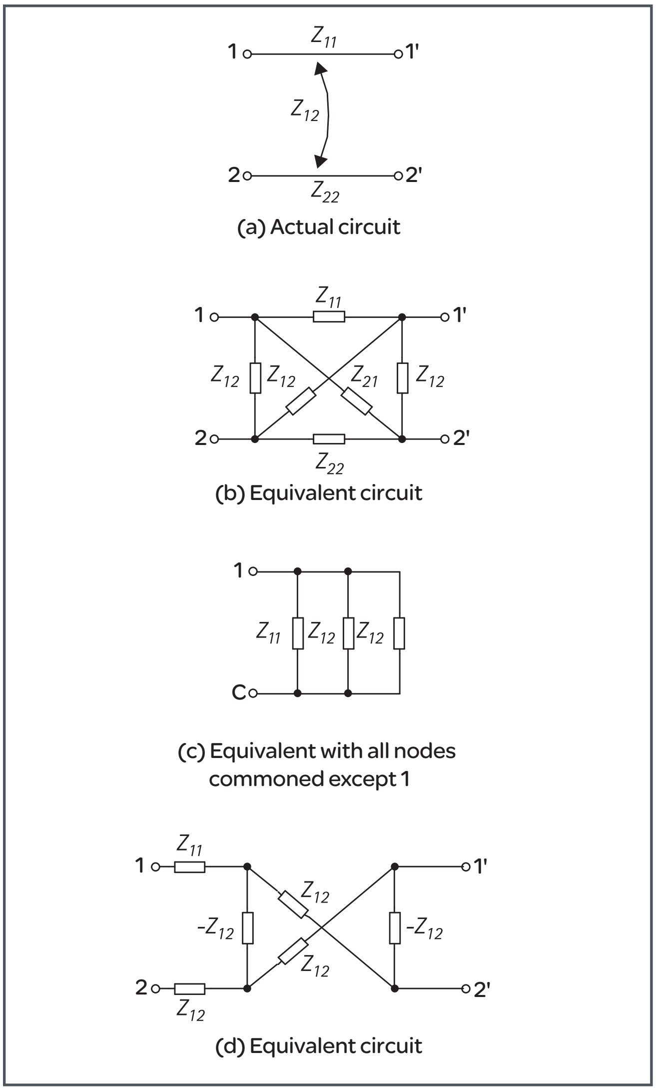

Figure A2.15 :
Equivalent circuits for four terminal network with mutual coupling

7. References

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